Generation of fast electrons by breaking of a laser-induced plasma wave

Raoul M. G. M. Trines,[1] Vladimir V. Goloviznin,[1,2] Leon P. J. Kamp,[1] and Theo J. Schep[2]

[1]*Technische Universiteit Eindhoven, P.O. Box 513, 5600 MB Eindhoven, The Netherlands*
[2]*FOM Institute for Plasma Physics Rijnhuizen, Nieuwegein, The Netherlands*

A one-dimensional model for fast electron generation by an intense, nonevolving laser pulse propagating through an underdense plasma has been developed. Plasma wave breaking is considered to be the dominant mechanism behind this process, and wave breaking both in front of and behind the laser pulse is discussed. Fast electrons emerge as a short bunch, and the electrostatic field of this bunch is shown to limit self-consistently the amount of generated fast electrons.

I. INTRODUCTION

Fast particle generation, the production of multi-MeV electrons during the propagation of an intense (10^{19} W/cm^2) laser pulse through an underdense plasma, is an aspect of laser-plasma interaction that has recently attracted a lot of attention, in both theoretical and experimental circles. The effect has been observed in many experiments [1–6], as well as in numerical simulations [7,8].

At the heart of this phenomenon lies the capture of plasma electrons by the laser-induced wake wave. Once captured, these electrons are accelerated by the wakefield, just as in a laser wakefield accelerator. Breaking of the wake wave has been suggested as the dominant mechanism for electron capture [2], although Raman backscattering might play a role in preaccelerating a small fraction of the background plasma electrons, which are then more easily captured and accelerated by the wake wave [5,6,9]. Closer investigations of electron capture due to wave breaking can also be found in Refs. [10,11]. Electron capture in a plasma wave induced by a relativistic electron bunch has been studied in Ref. [12].

In this paper, we develop an analytical one-dimensional (1D) model for the behavior of plasma waves, driven by the ponderomotive potential of a short $[L \simeq O(\lambda_p)]$ intense laser pulse. We assume that the laser pulse envelope does not evolve during its propagation through the plasma, and study only the plasma response to the pulse; as a consequence of this, the laser pulse intensity will not increase due to instabilities. This assumption is motivated by the observation that very short laser pulses show only minor envelope modification during their propagation through a plasma [13,14], whereas longer pulses show significant pulse shape modification due to, among other things, Raman scattering. We explore several scenarios that lead to the breaking of the plasma wave, and derive the necessary conditions for this to happen. We show that the fast electrons emerge as a very short bunch, whose electric field will limit the amount of generated fast electrons in a self-consistent way. From this, we derive the maximal charge density of the bunch as a function of laser pulse intensity.

II. ONE-DIMENSIONAL NONLINEAR PLASMA WAVES

In this paper, we start from an infinitely extended, homogeneous plasma, through which a laser pulse propagates in the z direction. The laser and plasma frequencies are denoted by ω and ω_p, respectively, and the plasma is assumed to be underdense: $\omega_p < \omega$. We denote the group velocity of the laser pulse, which is at the same time the phase velocity of the wake wave, by v_φ, and the corresponding Lorentz factor by γ_φ.

The following model assumptions concerning the plasma are made: the plasma is cold and temperature effects are neglected; the plasma electrons are initially at rest; the plasma background density n_0 changes on a very large length scale, so n_0 will be considered a constant. Concerning the laser pulse, we assume that it is one-dimensional and circularly polarized, and that its envelope does not change as it propagates through the plasma; as a consequence, instabilities in the pulse evolution due to Raman scattering are not investigated here. Furthermore, we assume that γ_φ is sufficiently large so all the terms of $O(1/\gamma_\varphi^2)$ or smaller can be neglected.

After separating fast and slow time scales, the slow, longitudinal motion of the electron fluid is governed by the following equations (the Coulomb gauge is used here):

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(n v_z) = 0, \tag{1}$$

$$\frac{\partial^2 \Phi_s}{\partial z^2} = -4\pi(n_0 - n)e, \tag{2}$$

$$\frac{\partial p_z}{\partial t} = \frac{\partial}{\partial z}(e\Phi_s - \gamma mc^2), \tag{3}$$

where n denotes the electron density, v_z and p_z the electron speed and momentum, $\gamma = \sqrt{1 + p_z^2/(mc)^2 + (e|A_\perp|)^2/(mc)^2}$ the Lorentz factor, $|A_\perp|$ the amplitude envelope of the laser pulse, and Φ_s the electrostatic wakefield potential. The other symbols have their usual meanings.

We apply the usual scaling transformations: $t' = \omega_p t$, $z' = (\omega_p/c)z$, $v' = v/c$, $p' = p/(mc)$, $\Phi_s' = e\Phi_s/(mc^2)$, $A_\perp' = eA_\perp/(mc)$, and drop the primes for convenience. In the scaled variables, $\gamma = \sqrt{1 + p_z^2 + U}$, where $U = A_\perp^2$ denotes the laser pulse envelope. We also introduce the new coordinates

$\zeta = z - v_\varphi t$, $\tau = t$. In line with our model assumptions, the wave is stationary in the frame moving with the pulse, i.e., $\partial/\partial\tau = 0$.

At this point, it is convenient to introduce the variable $\pi := \gamma - v_\varphi p_z$. As follows from the equations of motion, $\pi = 1 + \Phi_s$. From its definition, it follows that π has a U-dependent lower bound, $\pi \geq \pi_0(U) := (\sqrt{1+U})/\gamma_\varphi$ for all p_z. Furthermore, π reaches its lower bound for a certain value of p_z, $\pi = \pi_0(U)$ for $p_z = p_{cr} := \gamma_\varphi v_\varphi \sqrt{1+U}$. As a result of this, the mapping $\pi \mapsto p_z$ is double valued:

$$p_{z\pm} = \gamma_\varphi^2 [v_\varphi \pi \pm \sqrt{\pi^2 - (1+U)/\gamma_\varphi^2}], \qquad (4)$$

where $p_{z-} < p_{cr}$ and $p_{z+} > p_{cr}$. Since we wish to study regular plasma waves first, where the particle velocity is below the wave velocity, we take $p_z = p_{z-}$ for the moment.

After substituting $v_z = p_z(\pi)/\gamma$, the equation for π takes the form

$$\frac{d^2\pi}{d\zeta^2} = -\frac{\partial}{\partial\pi}V(\pi, U),$$

$$V(\pi, U) = \gamma_\varphi^2 [\pi - v_\varphi \sqrt{\pi^2 - (1+U)/\gamma_\varphi^2}]. \qquad (5)$$

This differential equation describes anharmonic oscillations of the variable π. (See also Ref. [15] for a similar equation, although with a slightly different $V(\pi, U)$, or Ref. [16] for a similar equation with ion motion included.) It is analogous to the Newton equation for a virtual particle with "coordinate" π, moving in a 1D "potential well" $V(\pi, U)$, where ζ is the "time" and $U(\zeta)$ acts as a "time"-dependent parameter. Continuing this analogy, we define the "velocity" of the virtual particle as $d\pi/d\zeta$, and its "kinetic energy" as $\frac{1}{2}(d\pi/d\zeta)^2$. Although somewhat artificial, the description of plasma oscillations in terms of oscillations of a virtual particle will provide much insight in this case, especially when studying necessary conditions for the occurrence of wave breaking.

It should be noted that a maximum (minimum) of π corresponds to a minimum (maximum) of the local electron density, with minimal (maximal) electron fluid momentum p_z and *zero* electric field. The minimum of $V(\pi, U)$ as a function of π corresponds to a point with maximal $|\partial\pi/\partial\zeta|$, i.e., maximal electric field, *zero* electron fluid momentum p_z, and $n = n_0$.

When $\pi \downarrow \pi_0(U)$, we have $\partial V/\partial\pi \to -\infty$, $n \to \infty$, and $p_{z-} = p_{z+} = p_{cr}$. It is possible that, after passing this point, $p_z = p_{z+}$ for some plasma electrons, so $\pi = \pi_0(U)$ is a bifurcation point. These electrons then overtake the wake wave, which is called wave breaking [19,20]. It can be concluded from the equations of motion that wave breaking is bound to happen if a sufficiently large oscillation is excited in the plasma.

III. EXCITATION OF 1D PLASMA WAVES

Following the approach of the previous section, longitudinal plasma waves are described as oscillations of a virtual particle with "coordinate" $\pi(\zeta)$, in a potential well
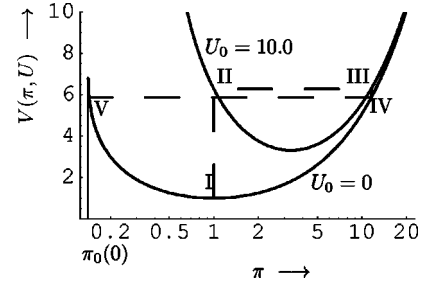

FIG. 1. Excitation of a plasma oscillation by a rectangular pulse with $n/n_{cr} = 0.02$ and $U_0 = 10.0$.

$V(\pi, U)$. The mechanism behind the excitation of such oscillations by strong laser pulses will be considered below.

For simplicity, we assume the laser intensity envelope $U(\zeta)$ is rectangular:

$$U(\zeta) = \begin{cases} U_0, & -L \leq \zeta \leq 0 \\ 0 & \text{otherwise.} \end{cases} \qquad (6)$$

This allows us to write down the energy equation for the virtual particle:

$$\frac{\partial}{\partial\zeta}\left[\frac{1}{2}\left(\frac{\partial\pi}{\partial\zeta}\right)^2 + V(\pi, U)\right] = 0, \qquad (7)$$

where one can see $V(\pi, U)$ as a function of π in two parameter regimes, $U = 0$ and $U = U_0$. Oscillations are excited by moving the virtual particle away from the bottom of the "potential well," located at $\pi = \pi_m := \sqrt{1+U}$. This will be done as follows (roman numbers correspond to those in Fig. 1).

(I) In front of the pulse, the plasma is at rest, so we start at $\pi = 1 = \pi_m$ for $U = 0$.

(II) At $\zeta = 0$, U increases to U_0, while π remains constant. Since the minimum of $V(\pi, U_0)$ is at $\pi = \sqrt{1+U_0} > 1$, the virtual particle will start to oscillate between $\pi = 1$ and $\pi = \pi_{II}(U) > 1$, where $\pi_{II}(U)$ denotes the other solution to $V(\pi, U) = V(1, U)$, and is given by

$$\pi_{II}(U) = \gamma_\varphi^2\left(2 - \frac{1}{\gamma_\varphi^2} - 2v_\varphi\sqrt{1 - (1+U)/\gamma_\varphi^2}\right). \qquad (8)$$

(III) When the laser intensity drops back to zero at $\zeta = -L$, the potential energy of the virtual particle drops from $V(\pi(L), U_0)$ to $V(\pi(L), 0)$.

(IV,V) Since $(\partial\pi/\partial\zeta)^2 > 0$ and $\pi > 1$, an oscillation in the potential well for $U = 0$ has been excited.

The total "energy" of the oscillation is given by $V(1, U_0) - V(1, 0) - (V(\pi(L), U_0) - V(\pi(L), 0))$, and since $V(\pi, U_0) - V(\pi, 0)$ decreases for increasing π, we find that the total oscillatory "energy" at a given pulse intensity U_0 is maximal for $\pi(L) = \pi_{II}$. For $\pi(L) < \pi_{II}$, the total "energy" differs from the maximal "energy" as $\Delta E \sim O([\pi_{II} - \pi(L)]^2)$.

One must note that, whereas the front edge of the laser pulse increases the potential energy of the virtual particle, the back edge decreases it; in the special case that $\pi(L) = 1$, the final energy of the virtual particle will be zero, and

there will be no wake behind the pulse. For this reason, we define the pulse length L to be optimal if the excited plasma oscillation has the maximal possible energy for the pulse intensity U_0. For the remainder of this paper, L is always assumed to be optimal.

If the total ''energy'' level of the oscillation after passage of the pulse exceeds $V(\pi_0(0),0)=\gamma_\varphi$, then we will have $\pi(\zeta)=\pi_0(0)$ for some ζ, while $|\partial\pi/\partial\zeta|>0$. [This means that, in Fig. 1, the end point $\pi_0(0)$ will be reached.] As mentioned above, the wave breaks at this point. For the maximal electrostatic field associated with a wave on the verge of breaking, we find, using $\partial\pi/\partial\zeta=(\omega/\omega_p)E_z$ and $(\partial\pi/\partial\zeta)^2 \leqslant 2[V(\pi_0,0)-V(1,0)]$,

$$E_{max}=(\omega_p/\omega)\sqrt{2(\gamma_\varphi-1)}, \tag{9}$$

which coincides with the wave breaking limit obtained by Akhiezer and Polovin [18].

IV. FAST PARTICLE GENERATION AS WAVE BREAKING

Thus, we know that the wake wave driven by the laser pulse breaks if $\pi(\zeta)=\pi_0(U_0)$ at some point, causing the production of electrons with speed $v_z>v_\varphi$. These electrons may be captured and accelerated by the wake wave itself, attaining a final energy up to $2\gamma_\varphi^2-1$. This mechanism is believed to be responsible for fast particle generation by intense laser pulses [2]. We are going to investigate two possible scenarios for reaching the wave breaking point: (i) wave breaking at the front edge of the laser pulse, and (ii) wave breaking during the first plasma oscillation behind the pulse.

In the first scenario, the envelope $U(\zeta)$ increases from 0 to its maximum value U_0, while at the same time, π stays at its original value of 1. (See steps I and II in Fig. 1.) The corresponding value of $\pi_0(U)$ increases from $1/\gamma_\varphi$ to $(\sqrt{1+U_0})/\gamma_\varphi$. If $\pi_0(U)$ reaches the initial value of $\pi=1$, the singularity at $\pi=\pi_0(U)$ will be encountered right at the front edge of the pulse. The threshold for this mechanism of wave breaking is

$$U_0=\gamma_\varphi^2-1. \tag{10}$$

This scenario could dominate fast electron generation at high laser intensities, but it does not explain the intensity thresholds found in experiments and simulations, which are commonly one order of magnitude lower than predicted.

In the second scenario, U_0 is assumed to be below γ_φ^2-1, so there will be no wave breaking at the front edge of the pulse. Instead, a plasma wave will be excited that is strong enough to break behind the pulse. As shown in the previous section, this will happen as soon as the total energy of the oscillation exceeds γ_φ. This situation can be reached as follows. Define π_1 as the other solution for π of $V(\pi,0)=V(\pi_0,0)$:

$$\pi_1=\gamma_\varphi^2(2-1/\gamma_\varphi^2)\pi_0(0)=\gamma_\varphi(2-1/\gamma_\varphi^2). \tag{11}$$

Clearly, the energy of the system will reach the wave breaking threshold if at point III in Fig. 1 $\pi(L)\geqslant\pi_1$.

If L is optimal, i.e., $\pi(L)=\pi_{II}(U_0)$, wave breaking will occur if $\pi_{II}(U_0)\geqslant\pi_1$, or

$$U_0\geqslant 2(\gamma_\varphi-1)-\frac{\gamma_\varphi-1}{4\gamma_\varphi^2(\gamma_\varphi+1)}, \tag{12}$$

which, since γ_φ is assumed to be large, will be approximated by $U_0\geqslant 2(\gamma_\varphi-1)$.

For any $\gamma_\varphi>1$, we find that the wave breaking limit for U_0 in the second scenario is smaller than in the first scenario, so this second scenario is at least partly responsible for fast electron generation at intensities below γ_φ^2-1, and it might define the intensity threshold for the existence of this phenomenon.

As an example, we see that for $n_0/n_{cr}=0.02$ and laser wavelength $\lambda_0=800$ nm, the first scenario predicts wave breaking for $U_0\geqslant 49$, corresponding to a power threshold of 1.1×10^{20} W/cm^2. The second scenario predicts a power threshold of 2.6×10^{19} W/cm^2. Simulations by Nagashima *et al.* [17] predict a threshold of approximately 10^{19} W/cm^2 for this case. Malka *et al.* [4] conducted simulations with $n_0/n_{cr}=0.05$ and $\lambda_0=1$ μm, and found an intensity threshold of $10^{18}-10^{19}$ W/cm^2, where our first scenario predicts $P=2.7\times 10^{19}$ W/cm^2 and the second predicts $P=1.0\times 10^{19}$ W/cm^2.

One should also note that several other mechanisms exist that may cause fast electron generation, such as Raman backscattering (see Ref. [9]) and thermal effects (see Ref. [20]). The associated intensity thresholds are slightly lower than those derived above.

V. BEHAVIOR OF THE BROKEN WAVE

The simple one-fluid model discussed here is suitable for the description of the plasma wave only until it breaks. For the description of the plasma after wave breaking, the model needs to be extended as follows. The plasma electrons captured by the wake wave can ultimately reach an energy of $\gamma_f=2\gamma_\varphi^2-1$, and a corresponding velocity v_f with $v_\varphi<v_f<1$. However, both v_φ and v_f are close to 1; for example, if $\gamma_\varphi=7.1$, we have $1-v_\varphi=0.01$, and $1-v_f=5\times 10^{-5}$. This means that captured plasma electrons will remain virtually immobile with respect to the wakefield, and bunch up just in front of the wave breaking point. This process cannot be described by our simple one-fluid model, so we need to extend the model to include the effect of a very short bunch of captured plasma electrons, moving with the wakefield. In the extended model, the relative density perturbation caused by fast electrons is approximated by a ''sheet'' of charge, located at $\zeta=\zeta_0$, close to the value of ζ at which the wave broke (about half a plasma wavelength behind the back edge of the pulse). As we shall see, the charge density in the sheet will increase until a certain limit has been reached. Then a new stationary state will be reached, consisting of a laser pulse, a plasma wave in the wake of the sheet, a charge sheet of a certain density located in the wake wave, and a (regular)
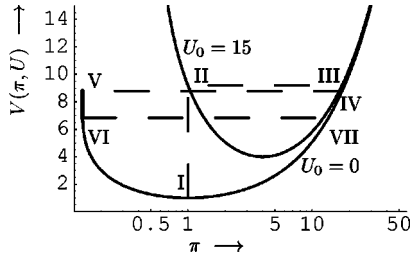
FIG. 2. The evolution of π and the total energy level of the system in the case of wave breaking and subsequent plasma electron capture. The laser and plasma parameters are $n/n_{cr}=0.02$ and $U_0=15$.

plasma wave in the wake of the sheet. This line of reasoning is supported by the simulation results of Nagashima *et al.* [17], which show a sharp spike in the electron density leaving the plasma and staying closely behind the laser pulse.

With the additional contribution of the charge sheet, the differential equation for π becomes [compare to Eq. (5)]:

$$\frac{\partial^2 \pi}{\partial \zeta^2} = -\frac{\partial}{\partial \pi} V(\pi,U) + Q\,\delta(\zeta-\zeta_0), \qquad (13)$$

where Q represents the saturated surface electron density of the sheet. We find that for both $\zeta>\zeta_0$ and $\zeta<\zeta_0$ the energy equation (7) still holds, while the presence of the sheet causes a jump in $\partial \pi/\partial \zeta$ at $\zeta=\zeta_0$, i.e., a jump in the electrostatic field. One should keep in mind that the extraction of fast electrons from the plasma leaves a positive charge behind, and the contributions to the electrostatic field of the fast electrons and the positive charge cancel in front of the pulse, so the presence of the charge sheet will not be felt for $\zeta > \zeta_0$.

The scenario of plasma wave excitation is depicted in Fig. 2.

(I–V) A plasma oscillation is excited similarly to the scenario without the charge sheet. Its energy level is higher than the energy $V(1/\gamma_\varphi,0)=\gamma_\varphi$ of the end point of the potential well.

(VI) After passage of the charge sheet at $\zeta=\zeta_0$, $\partial \pi/\partial \zeta$ has decreased by Q, and since $\partial \pi/\partial \zeta>0$ for $\zeta>\zeta_0$ we find that the "kinetic energy" of the virtual particle has also decreased. Since $\pi \approx \pi_0(0)$ at $\zeta=\zeta_0$, the "kinetic energy" just before passage of the charge sheet is given by $\frac{1}{2}(\partial \pi/\partial \zeta)^2 = V(\pi_{II}(U),0)-V(\pi_0(0),0)$. As long as Q is small, one still has $\partial \pi/\partial \zeta>0$ after passage of the charge sheet, and the growth of Q will continue; however, as soon as $Q^2 =2[V(\pi_{II}(U),0)-V(\pi_0,0)]$, fast electron generation, i.e., increase of Q, will stop.

(VI,VII) In the stable situation where the charge sheet prevents further wave breaking, there is a regular plasma oscillation behind the sheet.

From

$$V(\pi_0(0),0)=\gamma_\varphi, \qquad (14)$$

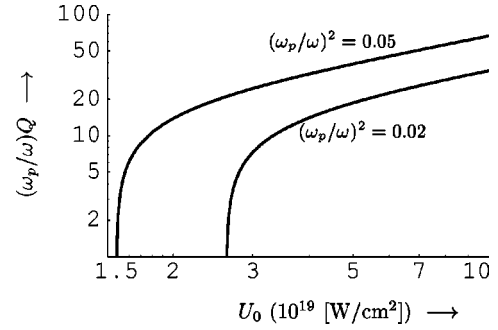$$V(\pi_{II}(U),0)=\gamma_\varphi^4(X-v_\varphi\sqrt{X^2-1/\gamma_\varphi^6}), \qquad (15)$$

FIG. 3. The total extracted charge density Q as a function of U_0 for $(\omega_p/\omega)^2=0.02,\ 0.05$.

where

$$X:=\frac{\pi_{II}}{\gamma_\varphi^2}=2-\frac{1}{\gamma_\varphi^2}-2v_\varphi\sqrt{1-\frac{1+U}{\gamma_\varphi^2}}, \qquad (16)$$

we find that the fast electron generation will stop as soon as

$$Q \approx \sqrt{\left(\gamma_\varphi^2+\frac{1}{4}\right)X+\frac{v_\varphi}{\gamma_\varphi^2 X}-2\gamma_\varphi},$$

$$\approx \sqrt{(U-U_{cr})+\frac{2(U-U_{cr})}{2+U_{cr}}+\frac{(U-U_{cr})^2}{(2+U_{cr})^2}}. \qquad (17)$$

Quite obviously, $U \geq U_{cr}=2(\gamma_\varphi-1)$ is the necessary condition for the formation of a charge sheet to happen. Since there will be no further charge buildup past this point, the above expression provides an upper limit for Q. The total surface charge density of the generated fast electrons will be $(n_0 e c/\omega_p)Q$. (See Fig. 3.)

The behavior of Q vs U_0 in Fig. 3 is in good qualitative agreement with simulation results by Nagashima *et al.* [17].

Note that U/γ_φ^2 should remain sufficiently small, otherwise wave breaking at the front of the pulse occurs, and the approximations made in the above equations are no longer valid.

VI. CONCLUSIONS

In summary, we developed a one-dimensional model for fast electron generation by a strong, nonevolving laser pulse propagating through an underdense plasma. Fast particles emerge as a result of wave breaking causing captured plasma electrons to bunch up just in front of the wave breaking point. The model describes the excitation of plasma oscillations by the ponderomotive force of the laser pulse, and provides intensity thresholds for the occurrence of wave breaking. It also includes, in a self-consistent way, the capture and acceleration of plasma electrons during wave breaking. The predicted intensity thresholds agree with well-known analytical results, and the amount of fast electrons as a function of laser intensity is in qualitative agreement with the results of recent particle-in-cell simulations [17].

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